

Questions and Cues

Notes

Moving Charges and Resulting Magnetic Fields in Conductors

These lectures are based on slides prepared by Dr. Danica Solina

Text: Walker et al. (2021), Halliday's Fundamentals of Physics – First Australian and New Zealand Edition
John Wiley & Sons Australia (HW)

Many thanks to Walter Kalceff whose notes formed the basic of these lectures....If I have seen further it is on the shoulders of giants!

Magnetism and Electromagnetism 1

1

The Magnetic Field:

Recall how the interaction of two charges was described through an electric field:

- one charge sets up an electric field \vec{E} in the space around it
- the electric field exerts a force $\vec{F}_E = q\vec{E}$ on a charge within the field.

Charges in motion:

- a moving charge (current) sets up a magnetic field in the space around it.
- the magnetic field exerts a force on a moving charge (current) in it.

Magnetic field is denoted by the symbol \vec{B} . It too is a vector quantity. The field is directed from North to South outside a magnet and has the units Tesla (T).

Magnetism and Electromagnetism 2

2

What force is experienced by a charge in a magnetic field?

It depends on:

- magnitude and sign of the moving charge, q (C)
- velocity of the charge, \vec{v} (m/s)
- magnitude of the magnetic field, \vec{B} (T).

The force (N) is given by:

$$\vec{F}_B = q \vec{v} \times \vec{B}$$

\vec{F}_B is always perpendicular to both \vec{v} and \vec{B}

It has magnitude:

$$F_B = qvB \sin \theta$$

where θ is the angle between the direction of the velocity and magnetic field.

We can combine this with the electric force to get the Lorentz force law:

$$\vec{F} = q \left(\vec{E} + (\vec{v} \times \vec{B}) \right)$$

PHYS 201 28.1 Magnetism and Electromagnetism 3

3

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Notes

Direction force on a charged particle in magnetic field

- The **right hand rule** can be used to find the direction of the force on a positive charge travelling in a magnetic field. The fingers point in the direction of the magnetic field, the thumb indicates direction of velocity of the POSITIVE particle and the palm the direction of the force on the particle. (or middle finger)
- The left hand can be used for NEGATIVE particles with fingers still pointing in the direction of the magnetic field and thumb the direction of the velocity of the NEGATIVE particle and the palm the direction of the resulting force on the particle.

What if the particle entered an extended uniform magnetic field always perpendicular to its path?

$q \vec{v} \times \vec{B} = \vec{F}_B$

$F = qvB \sin \theta$
 $\vec{F} \perp$ plane of \vec{v} and \vec{B}

<https://www.boundless.com/physics/textbooks/boundless-physics-textbook/magnetism-21/magnetic-force-on-a-moving-electric-charge-157/direction-of-the-magnetic-force-the-right-hand-rule-553-50931/>

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4

4

Charged Particle in Magnetic Field

Electron in Magnetic Field Proton in Magnetic Field

- Circle :if the motion is in a plane.
- Spiral with fixed radius if there is also a vertical component to \vec{v} .

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5

5

Magnetic Force

- It is the basis of many devices, including
 - ammeters, voltmeters
 - TV picture tubes (back in the day before plasma/LCD/OLED)
 - magnetic bottles for plasma containment in fusion
 - mass spectrometers
- Magnetic forces are difficult to imagine and illustrate, because \vec{F} is perpendicular to both \vec{B} and \vec{v} 😊

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6

6

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Notes

Units of \vec{B}

Given:

$$B = \frac{F}{qv}$$

the units of magnetic field are: $\text{NC}^{-1}\text{m}^{-1}\text{s} = \text{NA}^{-1} = \text{Tesla, T}$.
[In cgs units, B is in Gauss, G. 1 Tesla = 10^4 G.]

Typical Values for Magnetic Fields:
Earth’s field = several G (i.e. 10^{-4} T);
Strong permanent magnet ~ 0.4 T; large laboratory magnet ~ 1 – 3 T;
Superconducting magnet ~ 10 T; pulsed magnet up to 70 T.

PHYS 205, CH. 28.1

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7

7

Example

1. What is the magnetic force on an electron whose velocity is 2×10^6 m/s in a magnetic field of 2×10^{-2} T (200 G) directed perpendicular to the velocity?

2. How large an electric field would be required to produce a force of the same strength?

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8

8

Example

What is the force on an electron traveling at 3.5×10^6 m/s to the right in a magnetic field of 0.054 T out of the page at an angle of 45° to the field?

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9

9

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Notes

The mass spectrometer

- uses the Lorentz force- both electric and magnetic fields
- unknown molecule in vapour phase is ionised and dissociated by a high energy electron beam
- some ion fragments emerge through a small aperture into a uniform electric field between two parallel flat plate conductors with a large potential difference between them
- here they are accelerated to form a beam of ions which emerge through a second aperture, after which they pass through a uniform magnetic field perpendicular to their line of flight, before being detected.

The diagram illustrates the components of a mass spectrometer. On the left, a 'Cloud of ion fragments' is shown. These fragments pass through an aperture and are accelerated by an electric field between two 'Conducting plates' connected to a voltage source 'V'. The resulting 'Ion beam path' is shown as a straight line. After passing through a second aperture, the ions enter a region with a 'Uniform magnetic field' (indicated by 'X' marks). The magnetic field deflects the ions into a curved path, which is then detected by a 'Detector'.

10

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10

Analysis of mass spectrometer operation

On emerging from \vec{E} field, kinetic energy of the ionised fragments is qV . However,

$$KE = \frac{1}{2}mv^2 \quad \therefore \quad v = \sqrt{\frac{2qV}{m}}$$

where m is the mass of fragment, q is the charge on the fragment, V the potential difference between the plates.

Within the magnetic field region, \vec{B} and \vec{v} are mutually \perp , so

$$F = qvB = B \sqrt{\frac{2q^3V}{m}}$$

The magnetic force points "inward" and has a fixed magnitude...
Sounds rather like a centripetal force!

The diagram shows the same setup as the previous slide. It highlights the 'Ion beam path' as it enters the 'Uniform magnetic field' region, where it begins to curve towards the 'Detector'.

11

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11

Analysis of mass spectrometer operation

F is at right angles to both \vec{v} and \vec{B} , so the fragments must follow a circular path of radius r given by:

$$F_{mag} = \frac{mv^2}{r} = B \sqrt{\frac{2q^3V}{m}}$$

Hence radius of path taken by fragments is

$$r = \frac{1}{B} \sqrt{\frac{2mV}{q}}$$

Thus, by recording the position, r , where fragments are detected, their mass can be measured directly.

$qvB = \frac{mv^2}{r}$ is known as the Cyclotron formula

The diagram shows the 'Ion beam path' entering the 'Uniform magnetic field' region. The path is shown as a semi-circle that ends at the 'Detector'. The 'Conducting plates' and voltage source 'V' are also labeled.

12

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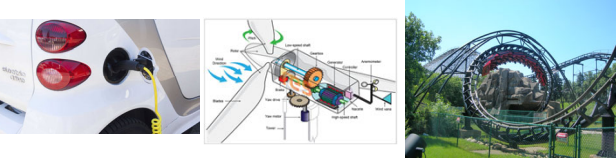
12

Questions and Cues

Notes

Other applications of Lorentz force

- **Electric motors** – moving charges (electric current) in a magnetic field create a force that turns the motor
- **Electric generators** – moving conductors (containing charges) in a magnetic field create an electromotive “force”
- **Electromagnetic braking systems** – conductive brakes move into a magnetic field creating torque on the charges and eddy currents

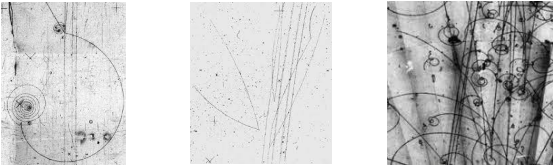


Magnetism and Electromagnetism 13

13

Other applications of Lorentz force

- Observation of fundamental particles in **bubble chambers**
 - tiny bubbles form along path of energetic charged particles, because molecules of supersaturated vapour become ionised and act as nucleation sites for bubbles
 - if the chamber is in a magnetic field, the direction of the tracks indicates the sign of the charge, while the radius is a measure of their momentum.



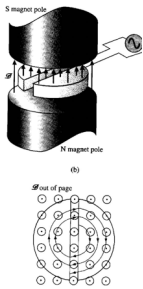
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14

Other applications of Lorentz force

Cyclotron (type of particle accelerator)

- Charged particles can be injected into the hollow “Dees” near their centre. A uniform magnetic field from the pole pieces of an electromagnet causes them to move in a circular path.
- As the particles come to the gap between the Dees, they are given a “kick” by a large E field. They accelerate and move into a larger radius orbit. The E field is arranged to reverse direction when the particles next cross it, again accelerating them.
- By the time they reach the edges of the Dees, the particles are ready to be extracted and used as required in various ways.



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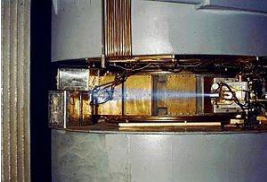
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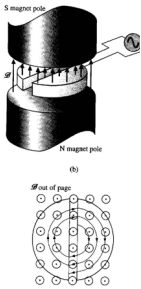
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Notes

Other applications of Lorentz force

Cyclotron (type of particle accelerator)
60-inch cyclotron, circa 1939, showing a beam of accelerated ions (likely protons or deuterons) escaping the accelerator and ionizing the surrounding air causing a blue glow





16

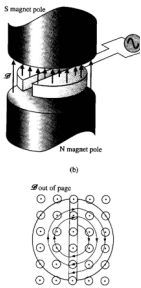
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16

Other applications of Lorentz force

Electron lenses (Lenses in electron microscopes)

- Blah blah



17

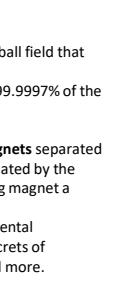
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17

Other applications of Lorentz force

Synchrotrons

- A synchrotron is a very large, circular instrument about the size of a football field that produces an intense beam of light (brighter than the sun!).
- Electrons are generated in the centre (electron gun) and accelerated to 99.9997% of the speed of light by the linear accelerator (electric field).
- The electrons are transferred to the outer storage ring.
- The electrons travel in a circle around the storage ring **by a series of magnets** separated by straight sections. As they are deflected through the magnetic field created by the magnets, they give off electromagnetic radiation, so that at each bending magnet a beam of synchrotron light is produced.
- The intense light produced is filtered and adjusted to travel into experimental workstations, where the light reveals the innermost, sub-microscopic secrets of materials under investigation, from human tissue to plants to metals and more.



18

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18

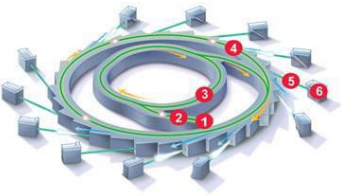
Questions and Cues

Notes

Other applications of Lorentz force

Synchrotrons


1. electron gun
2. linac
3. booster ring
4. storage ring
5. beamline
6. end station




19

Magnetism and Electromagnetism

Something rather curious

- Magnetic forces change the direction in which a charge is moving
- Magnetic forces cannot change the speed of the charge
$$\vec{F}_{mag} = q\vec{v} \times \vec{B}$$
- Magnetic forces do no work
 - If speed remains constant then KE remains constant and $W = \Delta K = 0$
 - Work done is energy transferred by application of a force along a displacement $W = \int \vec{F} \cdot d\vec{s} = 0$



20

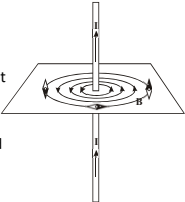
But if you can move something with a magnet isn't the magnetic field doing work??

A little histroy on Magnetism:

Oersted found the deflection of the compass needle is :

- Increased when the compass is placed closer to the current carrying wire.
- increased with larger current in the wire.
- unaffected by putting material (e.g. glass, wood, water and various rocks) between the wire and the compass.

A current carrying wire behaves like a magnet.



21

Assumed Knowledge Magnetism and Electromagnetism

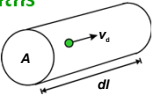
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Force on a current in a magnetic field – the maths

Recall current density:
$$\vec{j} = nq\vec{v}_d$$
where n is the number of charges per unit volume.
The current through A is
$$I = jA = nqv_dA$$

Suppose a wire of length $d\vec{l}$ is placed in a region of uniform magnetic field, \vec{B} , and current I is flowing in the wire (it's in a circuit!).
The total number of charge carriers in the bit of wire, moving at velocity v_d is
$$N = nAdl$$



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22

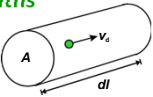
22

Force on a current in a magnetic field – the maths

The force on each charge is
$$\vec{F}_{mag} = q\vec{v}_d \times \vec{B}$$
Thus, the force on the all the charges is
$$d\vec{F} = nA(\vec{v}_d \times \vec{B})d\vec{l} = I(d\vec{l} \times \vec{B})$$

Note: $d\vec{l}$ is now a vector with the direction of **positive** charge flow.
When the angle between the field and wire is θ then the magnitude of the force is:
$$dF = I dl B \sin \theta$$

The electrons can't leave the wire but set up a little local electric field which acts on the positive nuclei, the wire itself experiences a force.



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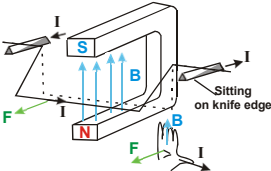
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23

23

Directions and Force:

What would happen if, in this diagram, we reversed
a) the current direction?
The force would be inward.
b) the magnetic poles?
The force would be inward.
c) both the current direction and the magnetic poles?
The force would be outward.



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24

24

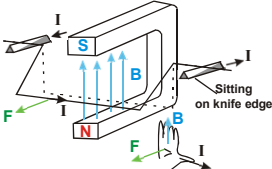
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Notes

Example

What force will a conductor of length 6.0 cm experience if it carries a current of 300 mA and is placed perpendicular to a magnetic field of strength 0.50 T as shown?

Solution



25

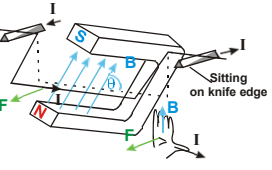
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25

Example

If we rotate our field in the previous example so that it is 60° to the conductor what force will the conductor now experience?

Solution



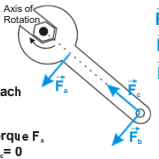
26

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26

Torque (τ in Nm)

Torque (or moment) of a force is a measure of its tendency to cause a body to rotate.



$|\vec{F}_1|=|\vec{F}_2|=|\vec{F}_3|$

but torque for each force is:

torque for $\vec{F}_1 >$ torque for \vec{F}_2 , and torque for $\vec{F}_3 = 0$

\vec{F}_1 : small effect

\vec{F}_2 : large effect

\vec{F}_3 : no effect

27

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27

Questions and Cues

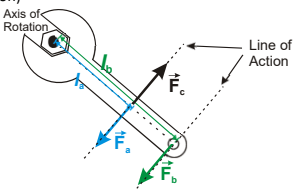
Torque (τ in Nm)

Torque is defined as the product of the force acting and the perpendicular distance, l , between the axis of rotation and the line of action.

$\tau_a = F_a l_a$ clockwise (-ve) ($\equiv -\hat{z}$ direction)

$\tau_c = F_c l_c$ counterclockwise (+ve)

The perpendicular distance, l , is also known as the 'lever arm' or 'moment arm'.



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28

28

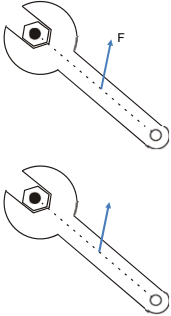
Torque (τ)

$\tau = Fl$

$= Fr_{\perp} \sin \phi$

$= F_{\perp} \sin \phi r$

$= \vec{r} \times \vec{F}$



Magnetism and Electromagnetism

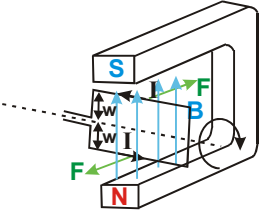
29

29

Sum m

What if.....

- instead we place n rectangular loops in our magnetic field?



$$F = \int dF = \int^L B I \sin \theta \, dl$$
$$= BIL \sin \theta$$
(if B is constant along L)

$$F = BIL \sin \theta$$
$$F = nBIL \sin \theta$$

Experiences Torque –
Rotates ->
The simple DC motor

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30

30

10

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Notes

DC Motor and Torque

The wire is always perpendicular to the field but, as it rotates, the torque it experiences changes as shown below:

I in

F

F

I out

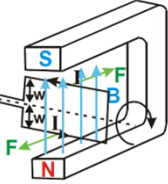
F perpendicular to moment arm
- Max Torque
 $\sin \varphi = \sin 90 = 1$

F

I in

I out

F parallel to moment arm
- Zero Torque
 $\sin \varphi = \sin 0 = 0$



Note: the loop rotates continuously in the field as it still has ω when torque is zero.

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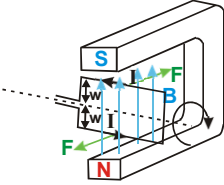
31

Calculating Torque

Using the equation for torque and substituting for force and the perpendicular distance from the rotation axis ' l ' with w :

$$\begin{aligned} \rho &= Fl \\ &= nBILw\cos\theta + nBILw\cos\theta \\ &= 2nBILw\cos\theta \\ &= nBIA\cos\theta \end{aligned}$$

Here θ is taken as the angle between the plane of our coil and the field and $A = 2Lw =$ area of coil in the field.



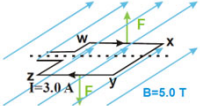
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32

Example

A rectangular coil (wxyz) with 150 turns lies with its plane parallel to a uniform magnetic field of strength 5.0 T as shown. A DC current of 3.0 A flows from a battery along the path wxyz.

- Show the direction of the force on the sides wx and yz.
- If the side xy is 30 mm long and the sides wx and zy are 50 mm, what are the forces acting on sides wx, xy and yz?
- What is the torque on the coil?



WU-CH-28.7 Magnetism and Electromagnetism 33

33

Questions and Cues

Notes

Example cont:

ii. If the side xy is 30 mm long and wx and zy are 50 mm, what are the forces acting on sides wx , xy and yz ?

iii. What is the torque on the coil?

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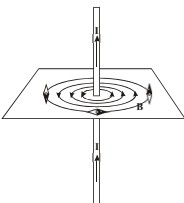
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34

34

Magnetic field of a straight conductor

Magnetic field lines are continuous. Unlike electric field lines, they do not originate at one location and end at another. The direction of magnetic field lines is defined by the right hand grasp rule. The thumb points along the wire in the direction of conventional current flow and the fingers curl around the wire in the direction of the magnetic field.

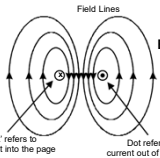


Density of lines: ?

Direction: ?

Field pattern of a current loop

Field Lines



'x' refers to current into the page

Dot refers to current out of the page

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35

35

The Biot-Savart Law

The Biot-Savart law allows magnetic fields to be determined near conductors. This law is the magnetic “equivalent” of the superposition principle. The law expresses the magnetic field as the sum of tiny sections of a current-carrying conductor.

The magnitude of the contribution to the magnetic field at point P due to a current element Idl (which points in the direction of current flow) is given by:

$$dB = \frac{\mu_0 Idl \sin \phi}{4\pi r^2}$$

Where μ_0 is a universal constant, called the permeability of free space and $\mu_0 = 4\pi \times 10^{-7} \text{ kgm/C}^2$ exactly.

PHYS 202

Magnetism and Electromagnetism

36

36

Questions and Cues

Notes

The Biot-Savart Law

In vector form:

$$d\vec{B}_P = \frac{\mu_0}{4\pi} \frac{Id\vec{l} \times \hat{r}}{r^2}$$

with r the displacement vector from $d\vec{l}$ to point P

The direction of \vec{B} is not along I or r but \perp (perpendicular) to a plane containing both I and r

The Biot-Savart (“bee-oh”) law allows the magnetic field allows the magnetic field \vec{B} to be calculated at any point in space due to a current in a complete circuit. It is given by the vector sum of all the contributions $d\vec{B}$ from the current elements $Id\vec{l}$. In integral form:

$$\vec{B}_P = \frac{\mu_0}{4\pi} I \int \frac{d\vec{l} \times \hat{r}}{r^2}$$

integrated along the length of current path

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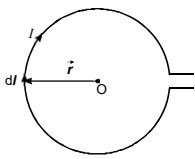
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37

37

Example

What is the magnetic field at the centre of a circular conducting loop?



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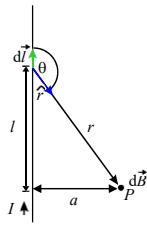
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38

38

Example

What is the magnetic field at a point P near a long, straight, current carrying wire?



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39

39

Questions and Cues

A little history on Magnetism:

- 1821: Andre-Marie Ampere (France), reasoned that if a current exerts a magnetic force on a compass needle, then two such wires should interact magnetically with each other. He showed in a series of experiments that
 - parallel (straight) currents attract
 - anti-parallel currents repel
 - the force is inversely proportional to the square of the distance between the wires.

Magnetism and Electromagnetism40

40

Notes

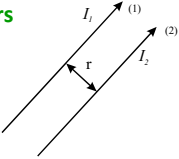
Force between long parallel conductors

We have seen that the force $d\vec{F}$ on a wire $d\vec{l}$ carrying a current I in a magnetic field \vec{B} is

$$d\vec{F} = I \, d\vec{l} \times \vec{B}$$

Now consider two long conductors separated by distance r and carrying currents I_1 and I_2 respectively. The magnetic field set up by (2) at the position of (1) is

$$B = \frac{\mu_0 I_2}{2\pi r}$$



The force on a length l of conductor (1) in this field is then

$$F = I_1 l \frac{\mu_0 I_2}{2\pi r}$$

Since $\vec{B} \perp l$ i.e.

$$F = \frac{\mu_0}{2\pi} \frac{I_2 I_1}{r}$$

Magnetism and Electromagnetism41

41

Basis of the definition of the Ampere:

An ampere is the current which must flow in each conductor for a force of 2×10^{-7} N to exist between them at a separation of 1m

$$F = \frac{\mu_0}{2\pi} \frac{I_2 I_1}{r} = \frac{4\pi \times 10^{-7}}{2\pi} \times \frac{1 \times 1 \times 1}{1} = 2 \times 10^{-7} \text{N}$$

Magnetism and Electromagnetism42

42

Summ

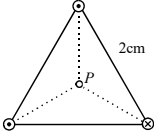
14

Questions and Cues

Notes

Example

Three infinite wires, each carrying a current of 5A, are arranged at the vertices of an equilateral triangle with 2 cm sides as shown below.



Calculate the magnitude and direction of the magnetic field at P , the centre of the triangle.

PHYS 202

Magnetism and Electromagnetism

43

43

Ampere's Law

Allows \vec{B} at a point near a current distribution to be calculated.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$$

In words:

The line integral of the magnetic field \vec{B} around any closed path is equal to μ_0 times the net current piercing the area bounded by the path.

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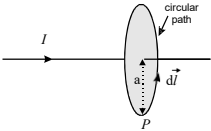
Magnetism and Electromagnetism

44

44

Example

What is the magnetic field near a long straight conductor?



PHYS 202

Magnetism and Electromagnetism

45

45

Questions and Cues

Notes

In applying Ampere’s Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

If \vec{B} is everywhere parallel (tangent) to the integration path, and $|\vec{B}|$ = constant at every point on the path, then the line integral is just $|\vec{B}|$ multiplied by the circumference of the path.

If \vec{B} is perpendicular to the path at any point(s), those parts of the path make no contribution to the line integral.

In $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$ B is the total magnetic field at each point of the path.

Choose the path carefully. The point(s) at which B is to be determined must lie on the path. The path must have enough symmetry to be evaluated.

PHYS 202

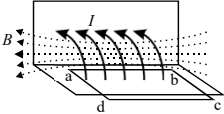
Magnetism and Electromagnetism

46

46

Example

What is the magnetic field at the centre of a solenoid?



PHYS 202

Magnetism and Electromagnetism

47

47
